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# The excitation statistics of disordered strongly correlated systems

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**Abstract.** The level statistics of a tight-binding Hamiltonian describing strongly interacting particles in a disordered system is investigated. It is found to be characterized by *two* transitions as functions of interaction strength. The first is from Poisson statistics to Wigner (GOE) statistics as interactions between the particles are turned on, and the second is back to Poisson statistics as the interactions become stronger. The dependences of both transitions on the interaction strength, filling factor, size of the sample, disorder and range of interactions are considered. Possible experimental consequences are discussed.

## 1. Introduction

Statistical properties of energy spectra of disordered quantum systems have been at the centre of much recent activity [1–11]. Part of the interest stems from the fact that they are related to localization of single-electron wave functions in disordered systems [2–5]. Most single-electron spectra can be described in the framework of random-matrix theory (RMT) originally proposed by Wigner and Dyson to explain the spectral properties of complex nuclei [12–14]. Metallic systems which have time-reversal symmetry obey the Gaussian orthogonal matrix ensemble (GOE) eigenvalue statistics, while for systems where time-reversal symmetry is broken (for example by a magnetic field) unitary matrix ensemble (GUE) eigenvalue statistics emerges. Symplectic ensemble (GSE) statistics is observed for cases in which spin–orbit scattering is present and time-reversal symmetry is respected. On the other hand, once the system is localized the spectrum follows Poisson statistics.

It has been shown [12, 13] that under the assumptions of RMT, i.e., for a system represented by a Hamiltonian of statistically independent random-matrix elements invariant under time reversal, the probability density of the level spacings is given by the Wigner (GOE) distribution

$$P_w(s) = \frac{\pi s}{2} \exp\left(-\frac{\pi s^2}{4}\right) \tag{1}$$

where $s$ is the energy separation between two consecutive levels in units of the mean level spacing $\Delta$. Another useful statistical measure is the variance of the number of levels in an energy window of size $E$ centred around a particular value of energy $\varepsilon$ denoted

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by $\mathrm{var}(N_\varepsilon(E)) = \langle N_\varepsilon^2(E)\rangle - \langle N_\varepsilon(E)\rangle^2$, where $\langle\ldots\rangle$ denotes an average over different realizations of disorder. Under the assumptions of RMT the variance is given by

$$\mathrm{var}(N_\varepsilon(E)) \sim \frac{2}{\pi^2}\ln(\langle N_\varepsilon(E)\rangle) + 0.44. \tag{2}$$

The RMT Gaussian ensemble corresponds to the statistical properties of a disordered system in the metallic regime as long as one refers to an energy window smaller than the Thouless energy $E_c$ (defined as $E_c = D/L^2$, where $D$ is the the diffusion constant and $L$ is the length of the system). For $E \gg E_c$ the variance behaves as [2]

$$\mathrm{var}(N_\varepsilon(E)) \sim N_d\left(\frac{E}{E_c}\right)^{d/2} \tag{3}$$

where $d$ is the dimensionality and $N_d$ is a numerical factor which for the three-dimensional case is $N_3 = \sqrt{2}/6\pi^3$ and for the two-dimensional case is $N_2 = 1/4\pi^2$.

For systems which may be described by a random Hamiltonian with almost zero off-diagonal elements the level spacing statistics corresponds to the Poisson distribution

$$P_p(s) = \exp(-s) \tag{4}$$

and the variance to

$$\mathrm{var}(N_\varepsilon(E)) \sim \langle N_\varepsilon(E)\rangle. \tag{5}$$

This is the case for a disordered system in the localized regime where the eigenvectors are spatially localized and have a very small overlap with each other.

The crossover between the various statistical ensembles at the metal–insulator transition is the focus of several recent studies [6]. The level separation at the transition is usually described in terms of a hybrid of the Wigner and Poisson distributions [6, 7, 8], which can be used to identify the critical disorder for which the transition occurs. Recently a new dimensionally dependent statistical behaviour of the spectrum at the mobility edge, different to the usual matrix ensemble results, was predicted [9]. Some evidence for that behaviour has been seen in a recent numerical study [10]. In another numerical study [11] it has been shown that the form of the distribution of the level spacing at the transition does not depend on whether the statistics in the metallic region was of the GOE or the GUE type.

Unlike for single-particle systems, the level statistics for interacting systems in condensed-matter physics is much less studied, although it is the standard starting point for nuclear physics. The many-particle spectrum of strongly correlated *ordered* condensed matter systems has been studied by Montambaux and co-workers [15, 16]. They have investigated the level separation statistics of the whole spectrum and shown that, except for in certain cases in which a system is integrable, the excitations follow a Wigner distribution.

On the other hand, the statistical properties of strongly correlated disordered systems have not been studied yet. In [17] the weakly interacting many-particle excitation spectrum of a disordered system was investigated. It was found that the high excitations of a non-interacting many-particle system follow Poisson statistics. This holds also for *very* weakly interacting systems. Once interactions become significant a transition of the many-particle excitation spectrum towards Wigner statistics is observed.

The strongly correlated disordered many-particle excitation spectrum of a quantum dot has recently become experimentally accessible by using a novel device composed of two quantum dots in a series in which one of the dots functions as a spectrometer and the differential conductance through the dot is measured [18]. This device has recently been used to measure the many-particle ground-state energy as function of the number of electrons

in the dot, and large fluctuations in this property were observed [19]. A similar technique may be applied to the study of the many-particle excitations in a quantum dot.

In this paper we shall investigate the Poisson to Wigner transition in detail and determine the strength of interaction for which this transition occurs. The level statistics follows Wigner statistics for an intermediate range of interaction strength. At stronger values of the interaction, a *second* transition occurs and the excitations again follow Poisson statistics. An explanation of this intricate behaviour of the many-particle level statistics will be provided based on the nature of the eigenvectors and the energy spectrum in different regions of parameter space.

We shall concentrate on the two statistical measures of the spectrum discussed above: the level spacing distribution and the variance in the number of levels in a given energy window. It will become apparent that special attention has to be given to the exact method of calculating the variance in order to avoid spurious fluctuations, the nature of which will be discussed below.

The paper is organized as follows: in section 2 a model of strongly correlated disordered many-particle systems is presented. In section 3 the behaviour of the level statistics for different filling factors and the form of the interaction as a function of weak electron–electron (e–e) interactions is discussed. The limit of strong e–e interactions is discussed in section 4. An extension of the results to large systems and a discussion of experimental relevance are presented in section 5. A comparison of the results to the level statistics of strongly interacting nuclear systems is also presented there.

## 2. The model

The strongly correlated disordered many-particle quantum dot will be modelled by a tight-binding 2D model represented by the following Hamiltonian:

$$H = H_{on} + H_{hop} + H_{int} \tag{6}$$

where

$$H_{on} = \sum_i \epsilon_i a_i^\dagger a_i \tag{7}$$

($\epsilon_i$ is the on-site energy which is chosen randomly between $-W/2$ and $W/2$), and

$$H_{hop} = V \sum_{\{i,j\}} (a_i^\dagger a_j + \mathrm{HC}) \tag{8}$$

where $V$ is a constant hopping matrix element and $\{i, j\}$ denotes summation over nearest-neighbour sites; and finally $H_{int}$ can take the form of a long-range Coulomb interaction between the electrons given by

$$H_{int} = \epsilon_c \sum_{i \neq j} \frac{(a_i^\dagger a_i - K)(a_j^\dagger a_j - K)}{|\boldsymbol{r}_i - \boldsymbol{r}_j|/u} \tag{9}$$

where $K$ represents a positively charged jellium and is equal to the average electron density, $\boldsymbol{r}_i$ is the position of the site, $u$ is the distance between nearest-neighbour sites and $\epsilon_c$ measures the strength of the interaction. The interaction can also take the form of a nearest-neighbour interaction:

$$H_{int} = \epsilon_c \sum_{\{i,j\}} a_i^\dagger a_i a_j^\dagger a_j. \tag{10}$$

The many-particle energy levels for a finite-size system of $m$ sites and $n$ electrons are calculated by the exact numerical diagonalization of an $M \times M$ matrix, where $M = \binom{m}{n}$. Once the eigenvalues are obtained for many realizations of disorder (we typically consider 500 different realizations) the energy level spacing distribution around any particular excitation, as well as the variance in the number of levels in a given energy window, may be calculated. We typically consider models of $3 \times 4$ sites ($m = 12$) and up to $n = 6$ electrons. The disorder is usually chosen as $W = 5V$ for which the single-electron energy spectrum follows a Wigner (GOE) distribution [17].

## 3. Weak electron–electron interactions

We shall start our discussion on statistical properties of the energy spectrum for the case of a many-electron non-interacting system studied in [17]. For that case, any many-electron state may be represented by electrons occupying a specific set of single-electron energy levels. The first two many-electron excitations follow the single-electron level spacing distribution because they always involve the transition of a single electron. This breaks down for higher excitations since for those excitations the transition of more than one electron might be energetically favourable. The transition of electrons between two adjacent states may also involve some of the electrons going down to lower single-electron states. Thus, even in the case for which the underlying single-electron levels exhibit level repulsion, one may expect the disappearance of level repulsion between two neighbouring many-particle energy levels. This can be understood in terms of Shklovskii's argument for the appearance of level repulsion [6]. Consider a system which is described by a random diagonal Hamiltonian $H_0$ with eigenvalues $\varepsilon_1, \varepsilon_2, \ldots$ and eigenvectors $|\Psi_1\rangle, |\Psi_2\rangle, \ldots$. Now let us consider what happens to two close adjacent eigenvalues $\varepsilon_n \to \varepsilon_{n+1}$ when one includes a non-diagonal term $V$ in the Hamiltonian. In this case one can treat those levels as a two-level system [2] for which in first-order perturbation theory, the separation between the eigenvalues is given by

$$s_n = \sqrt{(\varepsilon_{n+1} - \varepsilon_n)^2 + |\langle \Psi_n | V | \Psi_{n+1} \rangle|^2} \tag{11}$$

i.e., close levels repel each other. For the disordered Anderson model, the on-site energy (equation (7)) plays the role of $H_0$ while the hopping term (equation (8)) is analogous to $V$. Since the hopping term is a single-electron operator, once the adjacent levels differ by more than one single-electron occupation then $\langle \Psi_n | V | \Psi_{n+1} \rangle = 0$ and to first order there is no additional level repulsion. Therefore, when several transitions of electrons between neighbouring levels occur, one expects that the level spacing distribution will return to its original Poissonian form.

This is confirmed numerically in figures 1 and 2 where the excitation statistics of high-lying excitations are presented. Averaging over the distribution of 50 neighbouring excitations at the middle of the spectrum was performed in order to enlarge the statistical ensemble. This helps to smooth out the distribution without changing its qualitative form. It can be seen that for the non-interacting case there is no level repulsion. For the half-filled case ($n = 6$) there is a perfect fit to a Poisson distribution. For the quarter-filled case ($n = 3$) the numerical curve is lower than the Poisson curve. This might be expected since there are only three electrons and a non-negligible proportion of neighbouring many-particle states involve only single-electron transitions.

Once interactions between the particles are taken into account our previous arguments are no longer valid. In the case of weak e–e interactions one can gain some insight into the level repulsion behaviour from the following considerations. Since the non-interacting

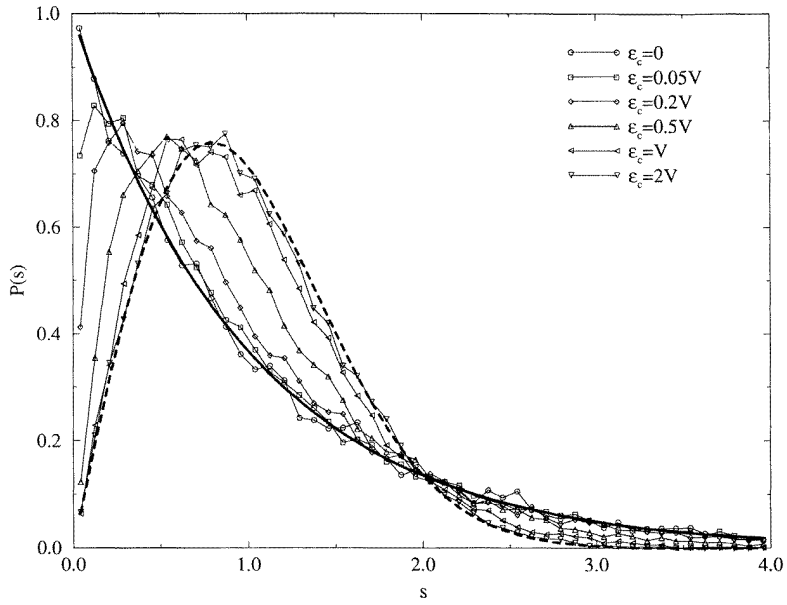


**Figure 1.** The many-particle level spacing $P(s)$ as a function of small values of the long-range e–e interactions for the half-filled case $n = 6$. The full curve represents the Poisson distribution (equation (4)) while the dashed line represents the Wigner (GOE) distribution (equation (1)).

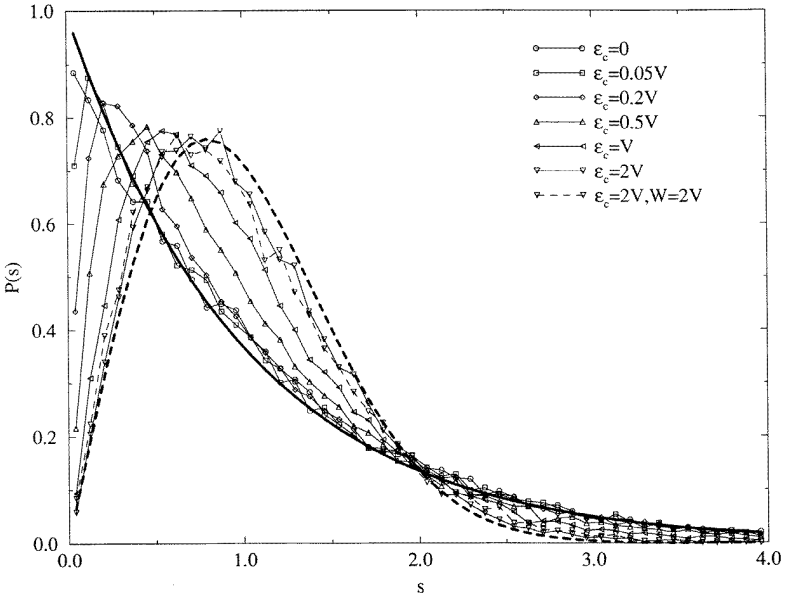


**Figure 2.** The many-particle level spacing $P(s)$ as a function of small values of the long-range e–e interactions for the quarter-filled case $n = 3$.

many-particle energy levels follow Poisson statistics, this system can be represented by a diagonal random Hamiltonian $H_{on} + H_{hop}$ with random diagonal terms $\epsilon_1, \epsilon_2, \ldots$, which

are of course the eigenvalues of the Hamiltonian and eigenvectors $|\psi_1\rangle$, $|\psi_2\rangle$, .... Now let us again consider what happens to two close adjacent eigenvalues $\epsilon_n \to \epsilon_{n+1}$ when one includes a small interaction term $H_{int}$ in the Hamiltonian. It is convenient to write $|\psi_n\rangle$ as a sum of eigenvectors in the site occupation number representation $|\varphi_i\rangle$ $(i = 1, \ldots, M)$

$$|\psi_n\rangle = \sum_i c_i^n |\varphi_i\rangle \tag{12}$$

where $c_i^n$ is a real amplitude. Thus the matrix element between the adjacent states is

$$\langle\psi_n|H_{int}|\psi_{n+1}\rangle = \sum_{i,j} c_i^n c_j^{n+1} \langle\varphi_i|H_{int}|\varphi_j\rangle \tag{13}$$

which, after inserting the explicit form of $H_{int}$ for the long-ranged interaction given in equation (9), gives

$$\langle\psi_n|H_{int}|\psi_{n+1}\rangle = \sum_i c_i^n c_i^{n+1} U_i \epsilon_c \tag{14}$$

where $U_i\epsilon_c$ is the electrostatic energy of the $|\varphi_i\rangle$ state given by

$$U_i = \sum_{k\neq l} \frac{\langle\varphi_i|(a_k^\dagger a_k - K)(a_l^\dagger a_l - K)|\varphi_i\rangle}{|\boldsymbol{r}_k - \boldsymbol{r}_l|/u}. \tag{15}$$

For high excitations one expects that the eigenfunctions $|\psi_n\rangle$ of $H_{on}+H_{hop}$ will be composed approximately equally from all the eigenvectors $|\varphi_i\rangle$ of the site representation, and therefore, because of normalization considerations ($\sum_{i=1}^M |c_i^n|^2 = 1$), $c_i^n \sim c_i^{n+1} \sim \pm 1/\sqrt{M}$. Summing $M$ random-signed contributions, while subtracting the average increase in the energy of each state due to the interactions, gives the following typical matrix element:

$$\langle\psi_n|H_{int}|\psi_{n+1}\rangle = \frac{\epsilon_c\,\Delta U}{\sqrt{M}} \tag{16}$$

where $\Delta U = \sqrt{\overline{U^2} - \bar{U}^2}$ and the average is performed over the $M$ different values of $U_i$. A numerical calculation of $\Delta U$ for several values of $m$ and $n$ is presented in figure 3.

The effect of the interaction on the level separation is expected to become significant once the matrix element is of the same order as the many-particle level spacing, which in the non-interacting metallic case can be crudely estimated as $\Delta \sim nV_{band}/M$, where the width of the single-electron band $V_{band} = 8V$. This estimation is reasonable as long as $W$ has no significant influence on the level spacing far from the tails of the band. Therefore, we expect a significant deviation from the Poissonian behaviour to occur at

$$\epsilon_c \sim \frac{nV_{band}}{\Delta U\sqrt{M}}. \tag{17}$$

Inserting the appropriate values of $V_{band}$, $n$, and $M$ in the previous equation, and using the values of $\Delta U$ for $m = 12$ and $n = 3$ or 6 appearing in figure 3, we end up with $\epsilon_c \sim 2.2V$ for the half-filled case and $\epsilon_c \sim 4.2V$ for the quarter-filled case. As can be seen in figures 1 and 2 even for a very weak interaction ($\epsilon_c = 0.05V$) signs of level repulsion already appear in the distribution of the level spacings. As the interaction becomes stronger the effect becomes more pronounced, and for $\epsilon_c \sim 2V$ the systems already show an almost perfect fit to the Wigner (GOE) level spacing distribution. This is in good agreement with our crude estimations for the half-filled case. For the quarter-filled case the transition occurs a bit earlier than calculated, which might be connected to the fact that in this case we did not have a fully developed Poisson distribution for $\epsilon_c = 0$. Nevertheless, it is interesting to note that for the half-filled case the level repulsion is stronger than for the quarter-filled case for the same value of $\epsilon_c$, as expected on the basis of our previous arguments.

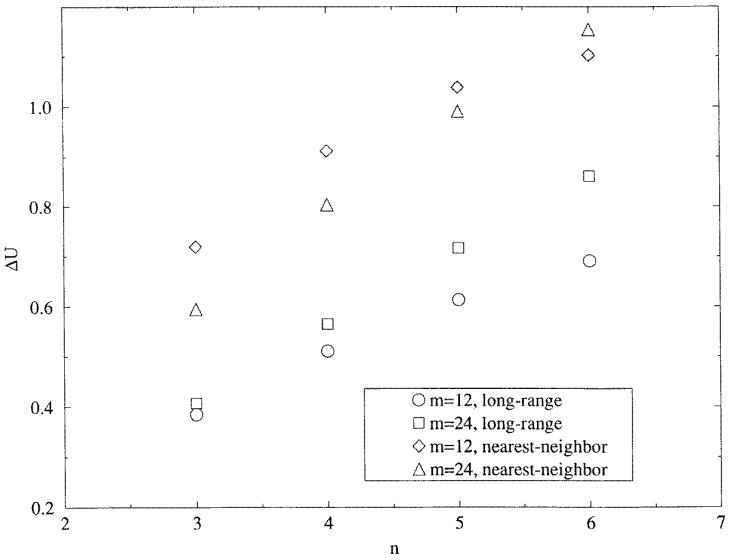


**Figure 3.** The fluctuations of the electrostatic energy $\Delta U = (\overline{U^2} - \bar{U}^2)^{1/2}$ where the average is performed numerically over the $M$ different values of $U_i$ for the appropriate type of interaction and for different values of $m$ and $n$.

As can be seen from equation (17) the transition does not depend explicitly on the strength of disorder. Thus, as long as the eigenvectors of the system in the site representation may be considered randomly mixed, i.e., $W$ is big enough for the mean free path to be shorter than the system length and smaller than the value for which the localization length is smaller than the system size, no dramatic change in the behaviour of the spectrum statistics is expected. This is verified in figure 2, where the level spacing distribution for $W = 2V$ is also plotted. No significant difference between the $W = 2V$ and $W = 5V$ cases is seen.

It is interesting also to compare the behaviour of the long-ranged Coulomb interaction to a nearest-neighbour interaction. The averaged electrostatic energy for the low-filling case can be easily estimated. The main contribution comes from states which have a single pair of electrons. Those states compose a fraction $Zn/m$ of all states, where $Z$ is the effective coordination number. Therefore, $\bar{U} \sim \epsilon_c(n/m)$, and $\Delta U \sim \epsilon_c(Zn/m)$. Thus, for the nearest-neighbour interaction the value of interaction for which a significant influence on the level spacing distribution is expected is of order $\epsilon_c \sim mV_{band}/Z\sqrt{M}$. The quarter-filled case cannot be considered a low-filling case since $Zn/m \sim 1$ and thus we will again obtain $\Delta U$ from the numerical calculation presented in figure 3, giving $\epsilon_c \sim 2.2V$, which is smaller than for the long-ranged Coulomb interaction. Thus, we expect that for the same value of $\epsilon_c$ the influence of the nearest-neighbour interactions on the energy level statistics will be larger than the long-ranged Coulomb interaction. This can be clearly seen in figure 4 where the level spacing distributions for several values of $\epsilon_c$ are presented.

As we have mentioned in the introduction another measure of the statistical properties of the energy spectrum is the variance in the number of many-particle energy levels for a given energy window. $\mathrm{var}(N_\varepsilon(E))$ as a function of $\langle N_\varepsilon(E)\rangle$ for the half-filled case is plotted in figure 5, where $\varepsilon$ is chosen as the averaged energy of the $M/2$ eigenvalue. It can be seen in the inset that for small values of $\langle N(E)\rangle < 4$ the $\epsilon_c = 0$ variance follows the Poisson result (equation (5)), while for $\epsilon_c = 2V$ the Wigner behaviour is observed (equation

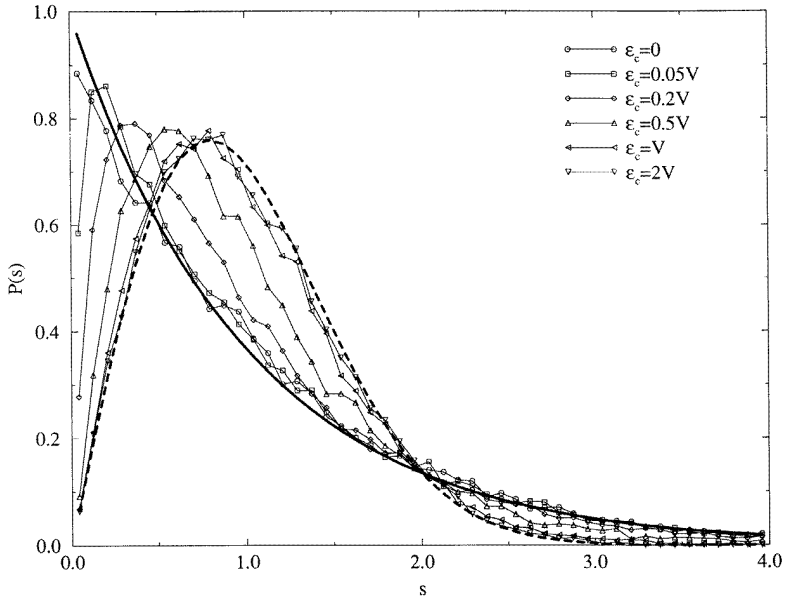


**Figure 4.** The many-particle level spacing $P(s)$ as function of small values of nearest-neighbour interactions for the quarter-filled case $n = 3$.

(2)). For values of $\epsilon_c$ in between a gradual crossover from the Poisson to the Wigner curve can be seen.

For larger values of $\langle N(E)\rangle$ a strong departure from both the Poisson and Wigner variance is apparent. Part of the reason for the large variance is sample to sample fluctuation. Small many-particle systems are especially sensitive to global displacements of the spectrum between realizations since there are only $m \ll M$ random site energies which determine all of the randomness in the spectrum. In order to see the net variance we also calculated the variance of the number of levels in an energy window of size $E$ centred around a particular energy level $\ell$ denoted by $\mathrm{var}(N_\ell(E))$. Thus the energy window is placed always around some particular energy level and is less sensitive to global displacements of the spectrum for different realizations of disorder. The results for $\mathrm{var}(N_{M/2}(E))$ are presented in figure 6. For $\langle N(E)\rangle < 10$ an excellent fit with the Poisson or Wigner prediction of the variance is evident. Also for higher values of $\langle N(E)\rangle$ the deviations from the predictions of Poisson or Wigner statistics are much smaller.

The same general behaviour can be seen also for the quarter-filled case. $\mathrm{var}(N_{M/2}(E))$ is plotted in figure 7. As for the distribution of level spacing, even for $\epsilon_c = 0$ the variances are below the Poisson prediction.

## 4. Strong electron–electron interactions

In the previous section we have shown that once the e–e interactions are strong enough the many-particle energy levels of an interacting system follow the Wigner (GOE) statistics. In this section we shall investigate what happens when the interactions are further increased, i.e., larger values of $\epsilon_c$ are assumed. It is convenient to begin by considering the limit $\epsilon_c \gg V$. In this limit one can treat the hopping term in the Hamiltonian ($H_{hop}$) as a

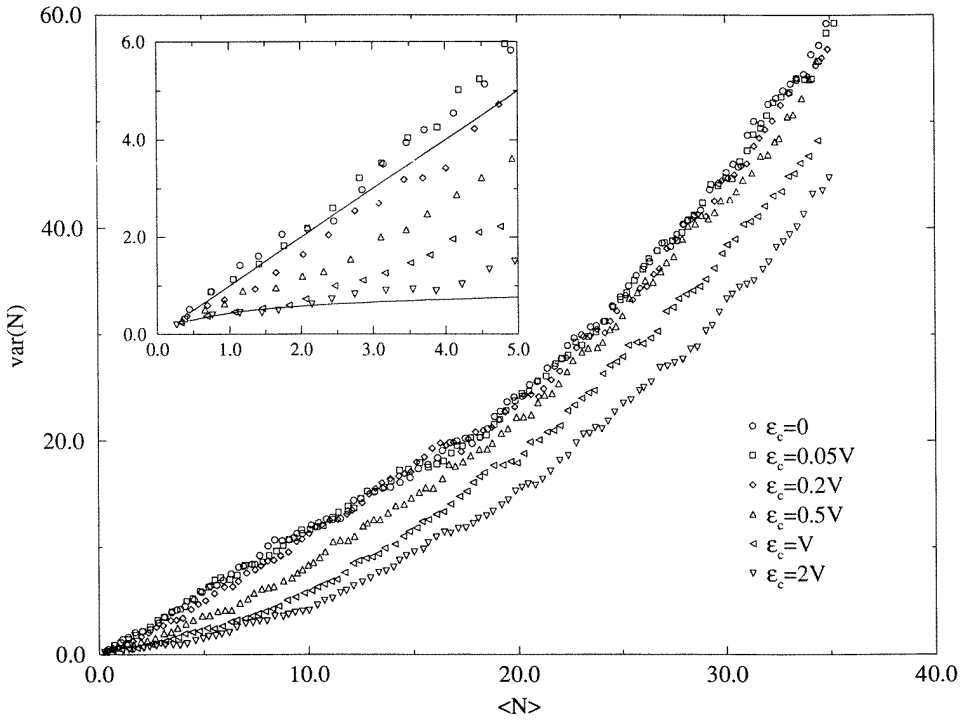


**Figure 5.** The variance in the number of many-particle energy levels var($N_\varepsilon(E)$) for a given energy window $E$ centred around an energy $\varepsilon$ as function of the averaged number of levels $\langle N_\varepsilon(E)\rangle$ for the half-filled case. The inset presents an enlargement of the $\langle N_\varepsilon(E)\rangle < 5$ region. The curves correspond to the theoretical predictions of the variance for Poisson and Wigner statistics.

perturbation to $H_{on} + H_{int}$ which is diagonal in the site representation. Therefore the $|\varphi_i\rangle$ are the eigenvectors of the unperturbed Hamiltonian with eigenvalues $\epsilon_i$. Since the on-site term in the Hamiltonian ($H_{on}$) is the only random part in the Hamiltonian and there are no off-diagonal terms one would expect the eigenvalue statistics to correspond to the Poisson statistics. This is verified in figures 8 and 9, and 11 and 12 below. Therefore, there must be a transition between the Wigner statistics which we have observed in the previous section for the energy level statistics for intermediate values of interaction to the Poisson statistics we expect for strong interactions.

Let us begin by quantifying the transition for the nearest-neighbour interactions. States in the site representation can be classified according to the number of nearest neighbours $p$. Thus, the energy of a state is given by its electrostatic energy $p\epsilon_c$ plus the influence of disorder. The disorder will change the energy of the state by an amount of order $\sqrt{n}W/2$. Therefore, strong interactions (i.e., $\epsilon_c > \sqrt{n}W/2$) will create 'bands' in the density of states of the Hamiltonian $H_{on} + H_{int}$. Once the bands are separate, each band can be treated independently. For states within a specific band $p$ the situation is very similar to the situation for the non-interacting case described in section 3. In order to estimate whether the statistics within the band corresponds to Poisson statistics or Wigner statistics we shall again use the first-order perturbation argument on the repulsion for two adjacent eigenvalues $\epsilon_n \to \epsilon_{n+1}$. When a small hopping term $H_{hop}$ is added to the Hamiltonian, the

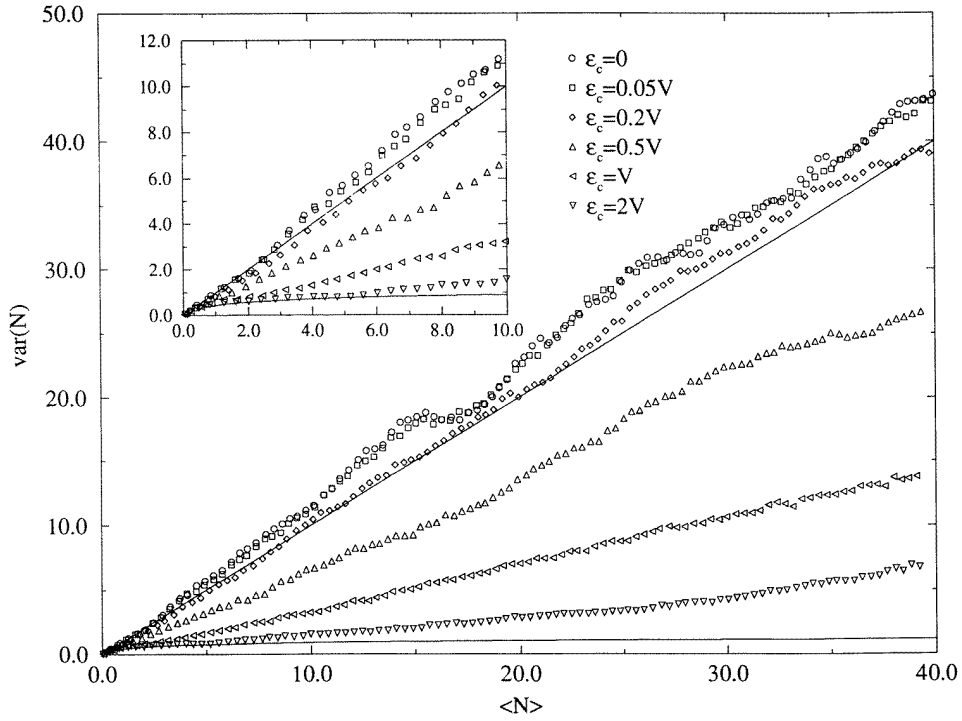


**Figure 6.** The variance in the number of many-particle energy levels var($N_\ell(E)$) for a given energy window $E$ centred around an energy level $\ell = M/2$ as function of the averaged number of levels $\langle N_\ell(E)\rangle$ for the half-filled case. The inset presents an enlargement of the $\langle N_\ell(E)\rangle < 10$ region.

level repulsion between the two states in the band is proportional to

$$\langle\varphi_n|H_{hop}|\varphi_{n+1}\rangle = \begin{cases} V & \text{if } |\varphi_n\rangle \text{ is different from } |\varphi_{n+1}\rangle \text{ by a single electron hop} \\ 0 & \text{otherwise.} \end{cases} \tag{18}$$

Thus, the value of the matrix element depends crucially on the degree of correlation between neighbouring eigenvectors. When there is no correlation, i.e., when states with $p$ nearest neighbours are different from each other by more than a single electron hop, the statistics should follow Poisson statistics since there is no repulsion. On the other hand, when states are different by a single electron hop, the statistics must remain Wigner statistics. For the half-filled case the 50 neighbouring levels at the middle of the spectrum belong to the $p = 4$ band. In this band there are only a few states which differ from each other by more than a single electron hop and therefore once $\epsilon_c > \sqrt{n}W/2 \sim 6V$ we expect to see only a small movement towards Poisson statistics which will not be enhanced as $\epsilon_c$ increases. This is confirmed in figure 8. An even more pronounced behaviour occurs for the quarter-filled case (figure 9). The 50 neighbouring levels at the middle of the spectrum belong to the $p = 1$ band for which two electrons are nearest neighbours and one has no neighbours. In this case many states differ by a single electron hop, resulting in no transition at all to the Poisson statistics even for extremely high values of $\epsilon_c$.

The general situation is somewhat similar for the long-range e–e interaction case, although the resulting level statistics behaviour will be totally different. High values of

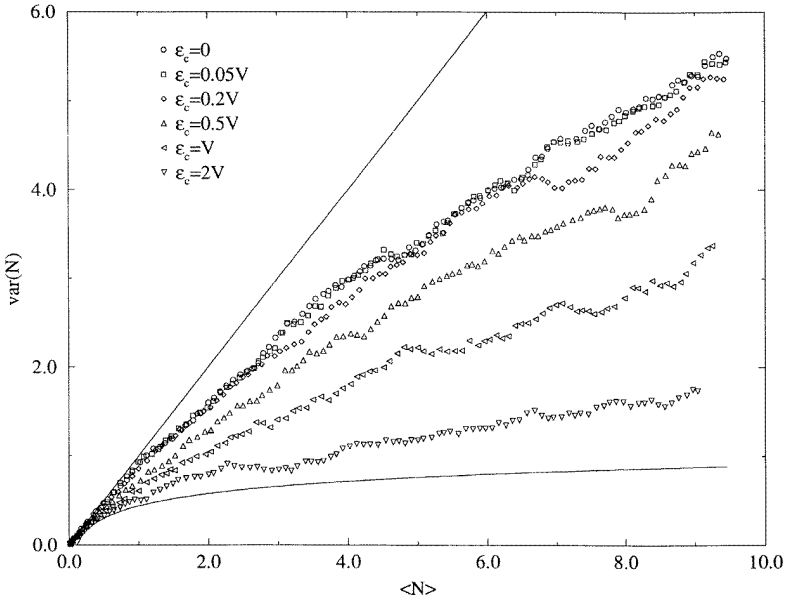


**Figure 7.** The variance in the number of many-particle energy levels var($N_\ell(E)$) for the quarter-filled case.

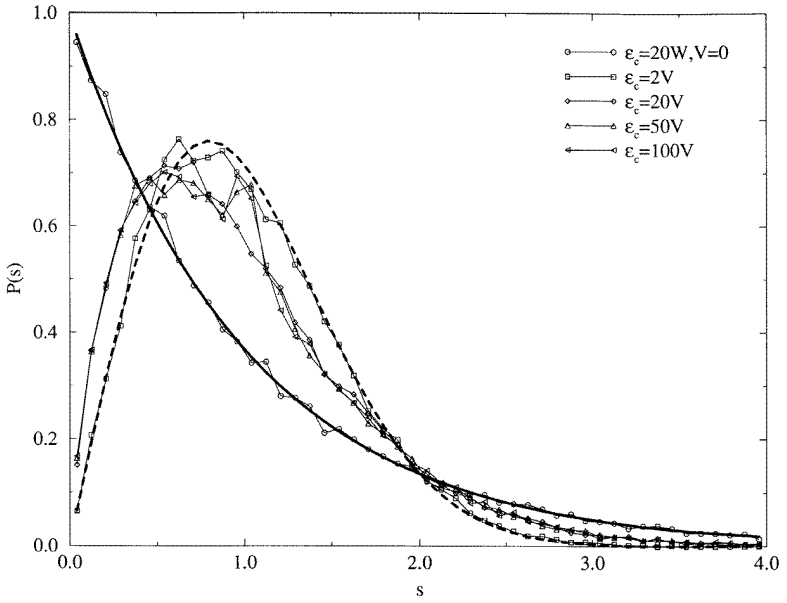


**Figure 8.** The many-particle level spacing $P(s)$ as function of large values of nearest-neighbour interactions for the half-filled case $n = 6$.

$\epsilon_c$ will create a series of bands, although the separation between the bands as well as the number of states per band strongly fluctuate (see figure 10). Another (more crucial) difference is that for the Coulomb interaction, states that have the same electrostatic energy

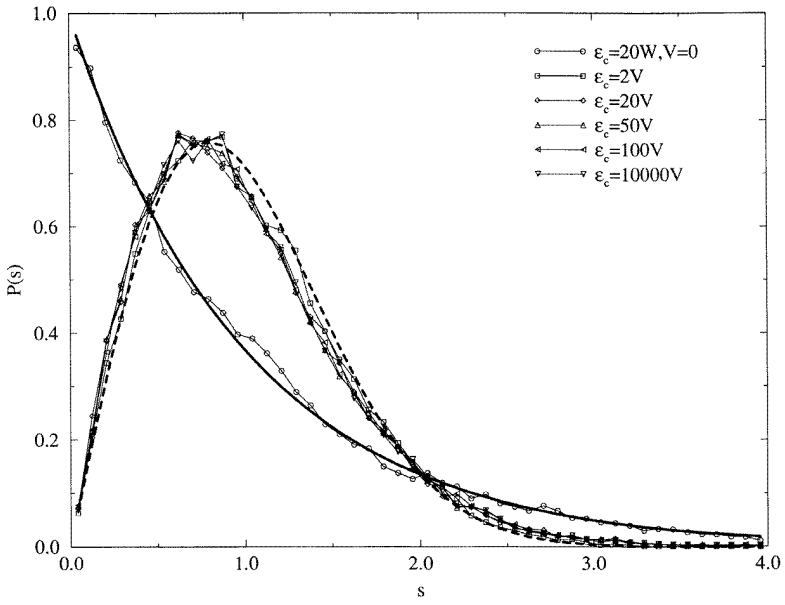


**Figure 9.** The many-particle level spacing $P(s)$ as function of large values of nearest-neighbour interactions for the quarter-filled case $n = 3$.

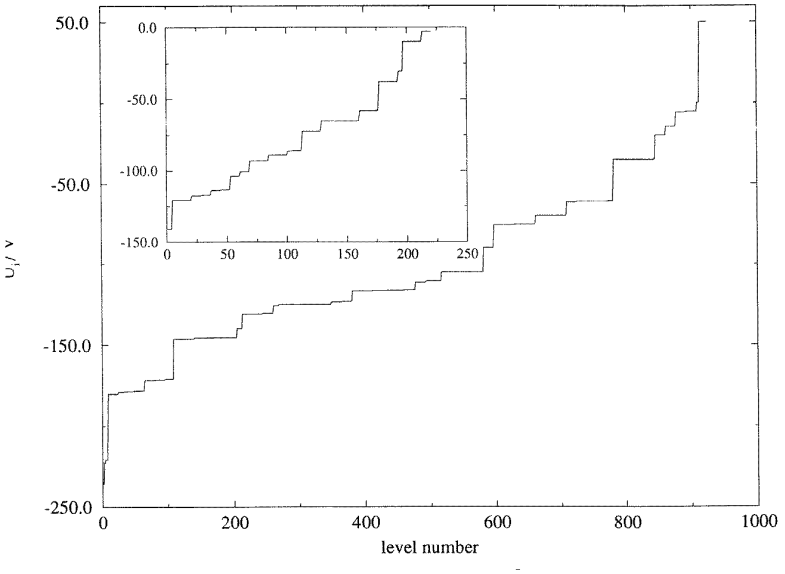


**Figure 10.** The electrostatic energy ($U_i = \langle\varphi_i|H_{int}|\varphi_i\rangle$) for the eigenvectors in the site representation arranged in an ascending order for the half-filled long-range e–e interaction at $\epsilon_c = 100V$. The inset presents the same data for the quarter-filled case.

generally differ from each other in the position of several electrons. Thus, we expect that once the bands separate, the statistics will change into Poisson statistics. From figure 10 it is clear that the typical energy separation between the bands around the middle of the

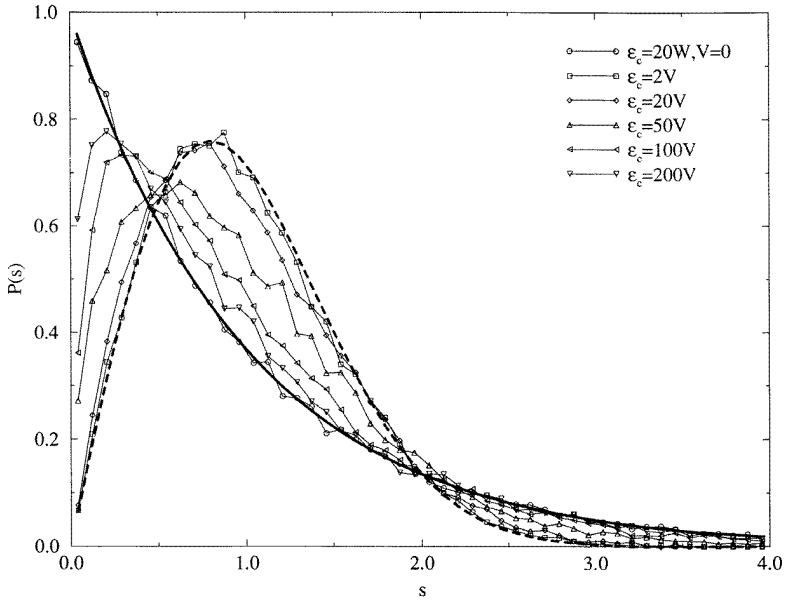


**Figure 11.** The many-particle level spacing $P(s)$ as a function of large values of long-range e–e interactions for the half-filled case $n = 6$.

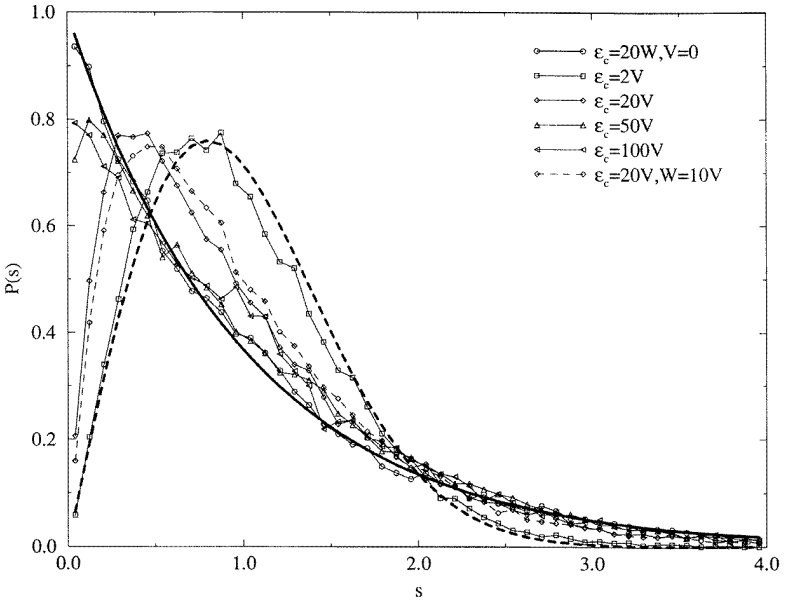


**Figure 12.** The many-particle level spacing $P(s)$ as a function of large values of long-range e–e interactions for the quarter-filled case $n = 3$.

spectrum is of order $0.05\epsilon_c$ for the half-filled case, and of order $0.1\epsilon_c$ for the quarter-filled case. We expect the transition to Poisson statistics to occur once the separation is larger than the width of the bands $\sqrt{n}W/2$. Therefore the transition is expected at $\epsilon_c \sim 120V$ for

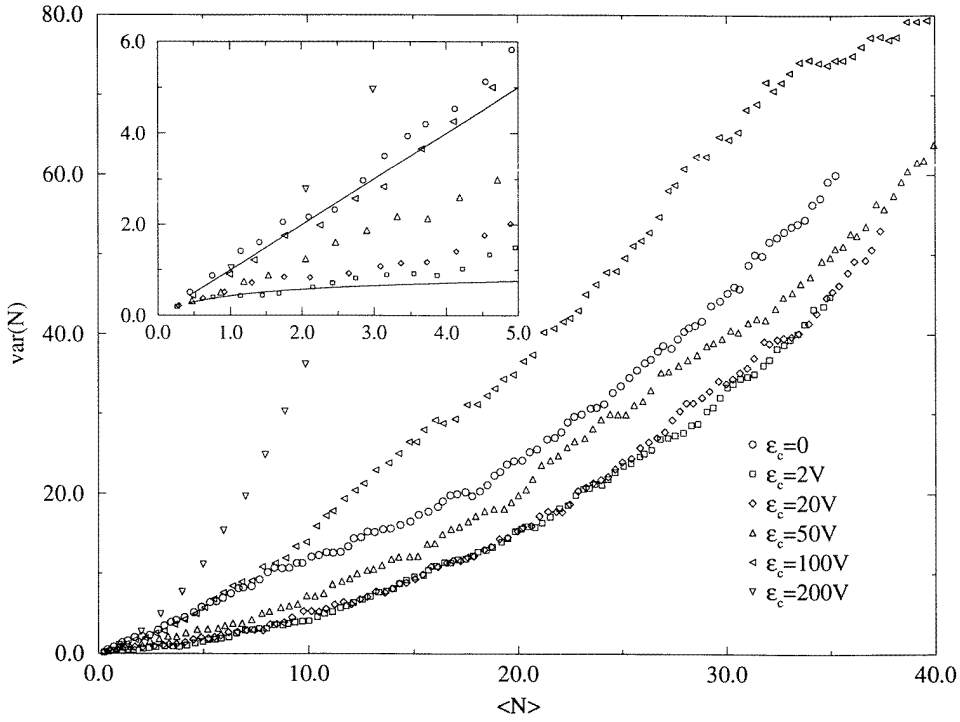


**Figure 13.** The variance in the number of many-particle energy levels var($N_\varepsilon(E)$) for strong long-range e–e interactions in the half-filled case. The inset presents an enlargement of the $\langle N_\ell(E)\rangle < 5$ region.

the half-filled case, and at $\epsilon_c \sim 40V$ for the quarter-filled case. This estimation corresponds rather well with the situation observed in figures 11 and 12.

From our arguments it is clear that the disorder $W$ plays a crucial role in determining the strength of interaction for which the transition will occur. This is checked in figure 12 where data pertaining to the $W = 10V$ case are presented. As expected, for the same value of interaction the $W = 10V$ case exhibits a stronger Wigner behaviour than the $W = 5V$ case.

As in the previous section we have also checked the variance var($N_\varepsilon(E)$) as a function of $\langle N_\varepsilon(E)\rangle$ for the half-filled Coulomb interaction case. It is plotted in figure 13, where again $\varepsilon$ is chosen as the averaged energy of the $M/2$ eigenvalue. One can see that the fluctuations, especially for strong interactions, are very high. Replacing var($N_\varepsilon(E)$) by var($N_\ell(E)$) we obtain the results presented in figure 14. It can be seen that the fluctuations are strongly reduced and the variance shows a clear transition between the Poisson and Wigner prediction of the variance, as expected from the level spacing distributions.

## 5. Discussion

In the previous sections we have seen that the energy level statistics of a disordered interacting many-particle tight-binding model undergoes two transitions as a function of the strength of the interaction between the particles. The first is a transition from the Poisson statistics which characterizes the energy spectrum of non-interacting many-particle

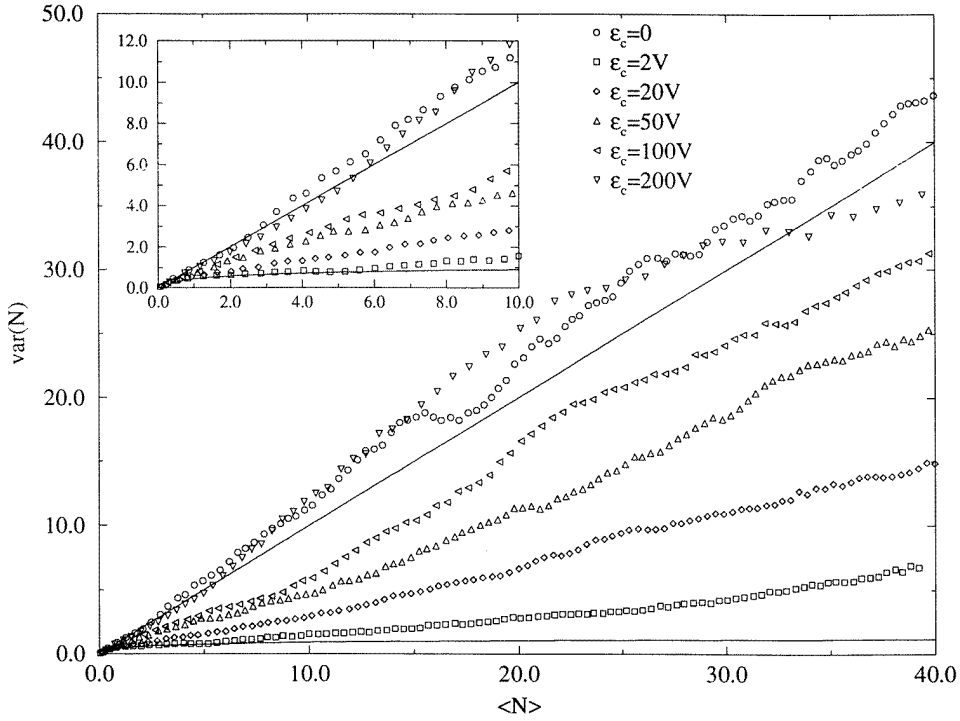


**Figure 14.** The variance in the number of many-particle energy levels $\mathrm{var}(N_\ell(E))$ for strong long-range e–e interactions in the half-filled case. The inset presents an enlargement of the $\langle N_\ell(E)\rangle < 10$ region.

systems to the Wigner (GOE) statistics. Assuming that the average electrostatic fluctuations depend only weakly (i.e., more weakly than exponentially) on the number of electrons and number of sites for a fixed filling factor, the dominant part in determining the value of interaction for which the transition occurs is the number of many-particle states, which rises exponentially as the system size is increased. Therefore, from equation (17), the strength of the interaction at which the transition occurs will decrease exponentially as the system size increases. This will result in Wigner statistics for high excitations of a large many-particle tight-binding model for even exponentially small interactions.

The second transition in the statistics of the spectrum is from Wigner (GOE) statistics of the levels back to Poisson statistics. This transition occurs once the interactions are strong enough to create non-overlapping bands in the system. It is not trivial to calculate the typical gaps in the spectrum of a classical ordered interacting system; nevertheless estimates show them to decrease slowly with the number of electrons or the number of sites. Therefore, for much larger systems than the ones considered here we expect that this transition will occur at a strength of interaction higher than for small systems, although not necessarily extremely high as long as the system is only weakly disordered.

Thus, the main chance of seeing a many-particle excitation spectrum which exhibits statistical properties different to those expected from RMT lies in examining small systems of a few interacting electrons which are relatively clean. This is exactly the situation in small quantum dots for which recent measurements of the ground-state energy were

performed [19].

Finally, we mention another example of strongly interacting many-particle Hamiltonians—the ones describing nuclear energy levels. For those systems it is well known that the experimental level statistics can be described rather well by the RMT of Gaussian ensembles [14]. The main difference between the two systems is that while the many-particle tight-binding model is described by a sparse matrix for which most matrix elements are equal to zero, in the nuclear Hamiltonian there are matrix elements of comparable size connecting all states. Therefore, in the nuclear Hamiltonian there is no single parameter (such as the interaction strength) which can change the behaviour from the RMT predictions to some other behaviour. Additional differences between the two systems are that nuclear Hamiltonians include attractive interactions and that they have no disorder (i.e., they are translationally and rotationally invariant). Thus the random nature of nuclear Hamiltonians originates from their complicated interactions, while in the tight-binding model it arises from the static impurity configuration.

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